“Recherches portant sur des télescopes pourvus d’une lame correctrice” “Research on telescopes with correction glasses

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Retyped in LaTeX, and attempted
French-to-English translation
by Matti Aarnio, 2001
INCOMPLETE – WORK IN PROGRESS

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Introduction of the translation

This translation is made in attempt to understand how professor Yrjö Väisälä did his wide-field Schmidt-Väisälä telescope, which carries special hallmark of *flat* optical focal plane, instead of the ball surface of the classical basic Schmidt telescopes.

The flatness of the focal plane is mandatory with modern electronic (e.g. CCD) sensors.

Without publishing it (he thought it was bad idea due to the focal surface being a non-flat one), he had thought of the Schmidt telescope idea long before of Bernhard Schmidt. Once Väisälä saw Schmidt's paper of 1931, he dug up his notes, and went on solving the "flattening of the focal surface" problem. But let Liisi Oterma tell the story below.

The motivation for doing this retype/translation results from wanting to understand Väisälä's Anastigmatic telescope (as he used to call it), which is now in use and care of our astronomy club.

Reasonably fittingly, Y.Väisälä was one of the founding members of our club in 1928, official registration is dated 12. January 1929.

We plan to modify this telescope a bit to use modern electronic sensors, but it was made for flat film (glass plate film!), and those didn't need more than 1 mm space in front of the film cassette, with focal plane being 3.0 mm inside the cassette, the distance from focal plane to field lens is something like 4-5 mm.

Making new focal field corrector at longer distance (20-30 mm away) is in our interest.

This dissertation contains clearest presentation of the design principles, which I have been able to find. (Besides of my zero knowledge of french language, which somewhat harms the understanding..)

The telescope we have was built in 1934, and got its "first light" in February 1935. (Sources appear to vary in detail, Oterma says that it became operational in fall of 1934.)

From 1935 to 1950es this telescope along with a creative photographic method was used to find over 800 minor planets, and a few score of comets.

During its peak, Turku University Observatory's search program was the most productive minor-planet hunt in the world with every two out of three minor planets being found coming from there. Much like the position which L.I.N.E.A.R. has now at the turn of the 21st century, some 60 years latter.

Maker of this dissertation, Doctor Liisi Oterma, was one of the principal people at doing that search work.

For translation I have employed (abused, some might say) “free” translation service by Altavista/Systran called “Babelfish” available on the Internet.
(http://www.altavista.com/)

The text is often verbatim copy of what Systran produces!

I have added some comments which appear as footnotes in the Oterma’s text.

As always with machine translations, some results are – lets say – hilarious, at such moments I may attempt to correct it a bit towards what I have understood the thing to mean. (E.g. “lame correctrice” Systran translates to “correct blade”, but in the context it is “corrector glass” – at least it is in our Finnish terminology.)

All translation/rewrite/english-grammar errors are all mine, I do know I make them (unintentionally, though.) I don’t indicate where raw Systran result is used, and what is “corrected”.

Matti Aarnio, Turun Ursa Astronomical Association

Introduction

For past few decades[1] large aperture telescopes have become more and more important for astronomy, and few related science fields. A thin corrector glass intended to remove the spherical aberration constitutes an essential part of these telescopes. This glass is usually made of one plane surface while the other has very slight rotation symmetric deviation from plane. The first published system of this type is known as SCHMIDT [1], which does not exhibit coma aberration. In this case the corrector glass is placed in the center of curvature of a spherical mirror. It is the system more used and SCHMIDT was the first to manufacture it. This otherwise excellent system has one inconvenience: the image-surface is a sphere. For significantly large view field the films and the photographic plates[2] must be deformed into spherical form, what can yield certain error terms where precise position measurement is of interest.

A curved field can be levelled by various methods. Simplest form is to use a field-corrector lens as proposed by PIAZZI-SMYTH in 1874. In the present case it is enough to place a convergent lens into proximity of the image-surface, and to change a little the distance from the corrector glass to the mirror and the profile of the corrector. The first telescope of this type is the anastigmatic telescope of the University of Turku, manufactured by Y.VÄISÄLÄ [2]. A simple aplanatic system with plane field will be obtained by placing a corrector glass close to the primary focus of ball-surface mirror [3, 4]. In this case the astigmatism will not be eliminated. The field will thus be narrower than if the curve of field is corrected by a lens but nevertheless the field will be much wider than in the case of a parabolic mirror. Moreover, the length of this system does not exceed the focal length.

The corrector glass can be replaced by a lens. or several lenses of sphreic surfaces. The first attempt of this type was probably done by A. KOURI[3] under the direction of Y. VÄISÄLÄ. A. KOURI replaced the corrector glass by two lenses, one convergent, the other divergent one, designed of the same type of glass. Such

1 1900 to 1955, presumably

2 there really used to be glass-plate film until about 1970es

3 A. KOURI: Korjauslasin korvaaminen linssisysteemillä aplanaattisessa teleskoopissa, 1938. [In Finnish] Unpublished work in masters thesis for the University of Turku sciences faculty.

combination does not present chromatic aberration.

Later a great number of various modifications intended for various uses were presented. For example HORST KÖHLER gave an outline of such in an article on the development of the aplanatic systems. [5]. It will be worthwhile to mention that KAY E. WEEDON collected quite excellent bibliography concerning the Schmidt system, and its modifications [6].

At the observatory of the University of Turku I carried out a plenty of research concerning the telescopes equipped with a corrector glass. Specific emphasis was on anastigmatic systems with field corrector lens. As the results of these are new in large part, it will be advisable to give a brief summary of what I did under the direction of Y. VÄISÄLÄ during these past years.

The idea to use a corrector glass to eliminate the spherical aberration of a mirror of telescope goes back to the year 1924. When preparing his lessons for the students, Y. VÄISÄLÄ drew into his pocket notebook diagrams of a number of telescopes utilizing a corrector glass. In these combinations the corrector glass is located in a general position, either in front of the mirror, or behind the mirror, i.e., between the mirror and the focal surface.[4] In this notebook a coma-free system is given as a particular example.

However the first practical experiment was carried out in 1928. In order to find of the conditions of making a corrector glass, we manufactured a corrector glass to correct the spherical aberration of a spherical mirror (one of 40 cm in diameter and 180 cm of focal distance). This corrector glass was placed in between the mirror, and the primary focus. The construction of this telescope was only provisional. It was only used for the study of the system. It gave clear images but the field of such a system is obviously small.

One could ask for the reason, why we made such a system. It was just so that we could only a small disc of optical glass.[5] We could have used, certainly, even a disc out of ordinary glass to make a decent quality mirror, but at that time of it, and even later the prejudice of the necessity of use of optical glass was prevalent. As the light goes thru the corrector glass, it was thought that the manufacturing of it must be of same perfection as in the case of an astronomical refractor lenses. To manufacture an astronomical refractive objective, one needs rather thick discs because the curves of the lenses forming it are relatively large. However in the case of the corrector glass things are different. The corrector is plane-parallel and can be quite thin. This way the changes in its form yield only small changes to optical paths. Y. VÄISÄLÄ did show one very interesting experiment: The corrector glass of the anastigmatic telescope (500/1031), referred to above [2], was replaced by another, a very thin one (8 mm thickness). Deforming this thin

[4] "Behind" here meaning "after visiting given optical surface", not physical position.
[5] Y. VÄISÄLÄ was also well known for penny-pinching.

corrector with a weight of 8 to 9 kg did not have photographically observable effect on the quality of the image.

Among our telescope constructions it is worth to mention the small test telescope (aperture of 172 mm, focal length of 344 mm) composed of a spherical mirror, a corrector glass, and a field corrector lens. The system, built during spring 1934, is the first in the world of this type. Immediately after this experiment we started to build larger telescope of this type (500/1031). It has flat focal field covering $6\frac{2}{3}$ degrees of arc. From autum of 1934 this telescope has been in use of observing minor planets, and comets. A bit smaller anastigmatic telescope (380/688) was build in 1939. It features focal field plane diameter of 10 degrees of arc.

Anastigmatic telescopes were built also for other observatories, and for other purposes (including aerial and maritime photography, and photographing X-ray fluorecent screens.) Presently a new test telescope is being built. It features corrector glass diameter of 0.90 meters, and has focal length of 2.50 meters. The spherical mirror is constructed of 7 smaller mirrors, which have special mounting construction [7, 8][6]

Original purpose of this dissertation was to present the methods and the formulas which were used in Turku for calculations of construction of the anastigmatic telescopes, but during work this task extended. I succeeded in improving somewhat the methods and the formulas, and found new questions. As there has not been available comprehensive presentation on this subject, I wanted to compose a kind of handbook which could be used for this type of research. This is why I decided also to deal with other problems having a close relationship with this research (parts I and II). Here a short summary on the contents of this work.

The 1st part gives a number of general formulas of geometric optics and the general symbols used in this work. Topics of the 2nd part relate to the systems made up of a mirror and a corrector glass. To start one dealt with the problem consisting in determining the profile of a corrector glass having a plane surface. I develop the methods to calculate the corrector glass profile up to 12th degree terms, which corrects the spherical aberration in the case the mirror surface is a generic rotational symmetric form, and where the distance from corrector glass to mirror is arbitrary.

Often mirrors are made so large that they do not obstruct of even considerably slanted light rays. The corrector glass and the mirror could be made of same diameters when a diaphragm is placed halfway in between them. Thus for same expense, a telescope with larger aperture can be made. For example Y. VÄISÄLÄ has applied this process while manufacturing anastigmatic telescopes. As the correct

[6]This telescope was the first attempt at a multiple-mirror system, but it didn't succeed due to excessive flexibility of the mirror support grid. The support structure is now at a museum.

lens of field does not have an essential action on the effect of the displacement of the diaphragm, one was satisfied to seek this effect by numerical calculations in a system not including/understanding a correct lens. For the calculation of the components of the transverse aberration one used the developments of the 5th order. For more perfection one also presented a simple establishment of the formulas by taking account of the possibility of small changes of the distance correct mirror-blade.

As the choice of the null deviation zone (zone by which the light rays parallel to the optical axis pass without being deviated) plays a significant role in the case of the corrector glass telescopes, and especially because one arrived at rather different results in this respect, one starts chapter 3 [XREF!] with research relating to this question. Then is shown that the null deviation zone can be moved further at an arbitrary distance by varying the curve of surface deformation. – I still study the effects caused by turning over, or curving the corrector glass. To simplify the calculations, I often assume that the deformed surface of the corrector glass is towards the mirror, other surface being plane. When such a system is constructed, photographic tests can't easily distinguish them from the primary images of stars. If one turns over surface deformation towards the mirror, this disadvantage will be reduced considerably. A means even more effective will be to curve the corrector glass.

Research relating to the anastigmatic telescopes is presented in the 3rd part. The anastigmatism was able to be removed with a field corrector lens. It seems to be a rather general use to level the field by a plano-convex lens, placed so that the image falls very close to the plane surface to the lens. This is why I initially sought this simple and theoretical case. It results from it that one can use, of course, a plano-convex lens as field corrector, but that this fact yields significant distortion. This distortion can be eliminated, like Y. VÄISÄLÄ proposed [2], by using a biconvex lens. To remove the coma which results from it, one must then slightly move the corrector glass towards the mirror and to remove the spherical aberration perfectly, the corrector glass must be deformed in a slightly different way than if the lens is plano-convex.

Then I give complete formulas and a numerical example to build, and control an anastigmatic system without a distortion, by supposing that the mirror is spherical. To search the possibilities of building systems of this type and to see what kind of degree of perfection is possible. These calculations show that one can remove fairly perfectly the other aberrations of the system, but what remains is a weak astigmatism which is growing with the increasing distance of corrector lens to the photographic plate (focal plane). This effect can be compensated partly by increasing the thickness of the lens. I also calculated small tables from which we can draw the approximate values from the curves of the lens.

Moreover one freely eligible size will be obtained if one gives up the sphericity of the mirror. This size will be used to relative remove the residue of the astigmatism at the zone of null deviation. Then one can use a little wider focal field. The formulas of construction, and control, in addition to the results, will be presented.

Still a freely eligible size will be obtained if one deforms also the field corrector lens. With a numerical example I show that this operation makes it possible to reduce the aberrations considerably.

Adding lenses into a system obviously causes chromatic aberration to appear. The null deviation zone should thus be moved. This question is treated in the last chapter. Calculations show that the chromatic aberration is always small in the corrector glass telescopes. Perhaps the importance of chromatic aberration has been exaggerated. The comparison made in between a corrector glass telescope, and an achromatic lens objective shows that especially if they are astronomical telescopes, i.e., of fairly large opening, the effect of the chromatic aberration is tens of times lower than that of secondary spectrum presented by a lens objective.

Part I

General formulas of geometrical optics for the co-axial systems

Let us start by giving a number of formulas of geometrical optics for the coaxial systems. These formulas are assumed to be well known but are presented here initially to show their form, and the symbols used in this work, and thus have them easily at hand.

Chapter 1

Symbols and Conventions

Figures [FIGREF1] and [FIGREF2] are used to illustrate the general symbols used in this work. In addition I introduce the following symbols:

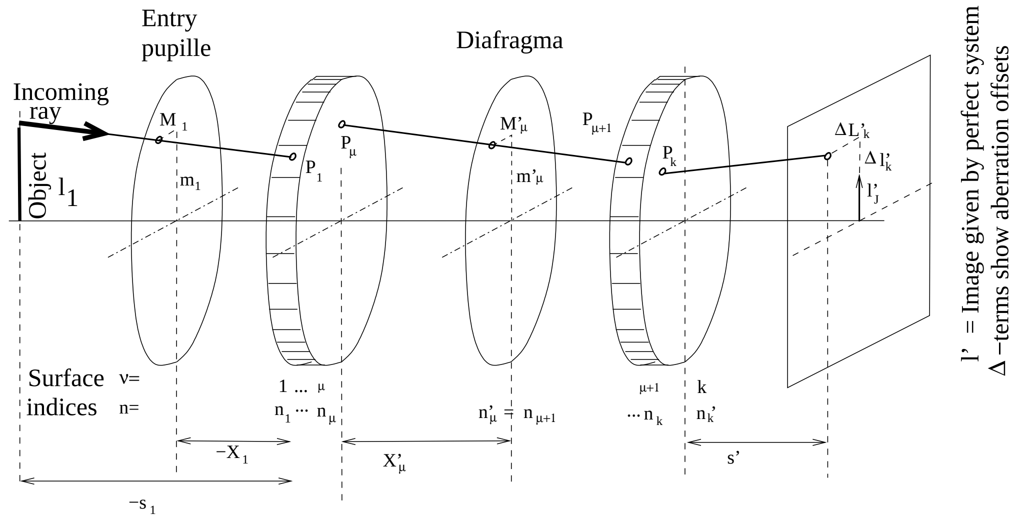
$$\varrho_\nu = \frac{1}{r_\nu};\ \sigma_\nu = \frac{1}{s_\nu};\ \sigma'_\nu = \frac{1}{s'_\nu};\ \varphi = \frac{1}{f} \qquad (f = \text{focal length})$$

$$v_\nu = -\tan u_\nu,$$

P_ν where the light ray bores surface ν,
h_ν distance of point P_ν from optical axle.
Profile of refractive (or reflective) surface:

$$g_\nu = g_\nu(h_\nu^2) = \frac{1}{2}\varrho_\nu h_\nu^2 + \frac{1}{8}h_\nu^4(\varrho_\nu^3 + \beta_\nu) + \cdots. \tag{1.1}$$

In the case of a spherical surface one will have: $\beta_\nu = 0$.



Chapter 2

Calculation of the behaviour of the paraxial rays

We calculate for $\nu = 1, 2, 3, \cdots, k$:

$$Q_{\nu s} = n_\nu(\varrho_\nu - \sigma_\nu) \qquad \sigma_{\nu+1} = \frac{\sigma_\nu^{'}}{1 - e_\nu^{'}\sigma_\nu^{'}}$$
$$\sigma_\nu^{'} = \varrho_\nu - \frac{1}{n_\nu^{'}}Q_{\nu s} \qquad \eta_{\nu+1} = \eta_\nu(1 - e_\nu^{'}\sigma_\nu^{'}). \tag{2.1}$$

In the particular case where $\sigma_1 = 0, n_1 = n'_k = 1$, we have:

$$\varphi'_k = \eta_k\sigma'_k = \sigma'_k\prod_{\nu=1}^{k-1}(1 - e'_\nu\sigma'_\nu). \tag{2.2}$$

In the case of the reflection we calculate as if it were about an infinitely thin lens whose other surface would be plane, and the index $n = -1$.

Chapter 3

Calculation of the aberrations of the 3rd degree

Calculation of the aberrations for each surface in turn.

$$\begin{aligned}
A_\nu &= \eta_\nu^4(\beta_\nu \Delta_\nu n + Q_{\nu s}^2 \Delta_\nu \frac{\sigma}{n}) \qquad {}^1\text{(For reflection: } \Delta_\nu n = -2\text{)} \\
B_\nu &= \delta_\nu A_\nu + \eta_\nu^2 Q_{\nu s} \Delta_\nu \frac{\sigma}{n} \\
C_\nu &= \delta_\nu^2 A_\nu + 2\delta_\nu \eta_\nu^2 Q_{\nu s} \Delta_\nu \frac{\sigma}{n} + \Delta_\nu \frac{\sigma}{n} \\
D_\nu &= \varrho_\nu \Delta_\nu \frac{1}{n} \\
E_\nu &= \delta_\nu^3 A_\nu + 3\delta_\nu^2 \eta_\nu^2 Q_{\nu s} \Delta_\nu \frac{\sigma}{n} \\
&\quad + \delta_\nu (3\Delta_\nu \frac{\sigma}{n} - \varrho_\nu \Delta_\nu \frac{1}{n}) \\
&\quad + \frac{1}{\eta_\nu^2 Q_{\nu s}} (\Delta_\nu \frac{\sigma}{n} - \varrho_\nu \Delta_\nu \frac{1}{n}),
\end{aligned} \tag{3.1}$$

where:

$$\delta_\nu = \sum_{\mu=2}^{\nu} \frac{e'_{\mu-1}}{n'_{\mu-1} \eta_{\mu-1} \eta_\mu} = \sum_{\mu=2}^{\nu} \lambda_{\mu-1}.$$

In particular, if surfaces are spherical, it is useful to state:

$$\varepsilon_\nu = \frac{1}{\eta_\nu^2 Q_{\nu s}}; \qquad \tau_\nu = \varepsilon_\nu,$$

[1] $\Delta_\nu \frac{\sigma}{n} = \frac{\sigma'_\nu}{n'_\nu} - \frac{\sigma_\nu}{n_\nu}$ etc.

which will give:

$$\begin{aligned} A_\nu &= \eta_\nu^4 Q_{\nu s}^2 \Delta_\nu \frac{\sigma}{n} \\ B_\nu &= \tau_\nu A_\nu \\ C_\nu &= \tau_\nu B_\nu \\ D_\nu &= \varrho_\nu \Delta_\nu \frac{1}{n} \\ E_\nu &= \tau_\nu (C_\nu - D_\nu) \end{aligned}$$

Seidel terms:

$$\begin{aligned} \mathsf{I} &= \sum A_\nu \\ \mathsf{II} &= \frac{1}{q_1} \sum A_\nu + \sum B_\nu \\ \mathsf{III} &= \frac{1}{q_1^2} \sum A_\nu + \frac{2}{q_1} \sum B_\nu + \sum C_\nu \\ \mathsf{IV} &= \sum D_\nu \\ \mathsf{V} &= \frac{1}{q_1^3} \sum A_\nu + \frac{3}{q_1^2} \sum B_\nu + \frac{1}{q_1} (3 \sum C_\nu - \sum D_\nu) + \sum E_\nu, \end{aligned} \tag{3.2}$$

where, in the case the object is at the infinity:

$$\frac{1}{q_1} = -X_\nu .$$

In general case:

$$\begin{aligned} \frac{1}{q_1} &= \frac{X'_{\mu-1}}{\eta_{\mu-1}^2 (X'_{\mu-1} \sigma'_{\mu-1} - 1)} - \delta_{\mu-1} \\ &= \frac{X_\mu}{\eta_\mu^2 (X_\mu \sigma_\mu - 1)} - \delta_\mu \end{aligned} \tag{3.3}$$

and conversely:

$$\begin{aligned} X'_{\mu-1} &= \frac{\eta_{\mu-1}^2 (\frac{1}{q_1} + \delta_{\mu-1})}{\sigma'_{\mu-1} \eta_{\mu-1}^2 (\frac{1}{q_1} + \delta_{\mu-1}) - 1} \\ X_\mu &= \frac{\eta_\mu^2 (\frac{1}{q_1} + \delta_\mu)}{\sigma_\mu \eta_\mu^2 (\frac{1}{q_1} + \delta_\mu) - 1} . \end{aligned}$$

Components of the transverse aberration, and curves of focal surfaces in the particular case where $s_1 = \infty$, $n_1 = n'_k = 1$:

$$
\begin{aligned}
2\frac{\Delta l'_k}{f} &= -m_1(m_1^2 + M_1^2)\,\mathsf{I} - (3m_1^2 + M_1^2)v_1\,\mathsf{II} \\
&\quad + m_1 v_1^2(\mathsf{IV} - 3\,\mathsf{III}) - v_1^3\,\mathsf{V} + \cdots \\
2\frac{\Delta L'_k}{f} &= -M_1(m_1^2 + M_1^2)\,\mathsf{I} - 2m_1 M_1 v_1\,\mathsf{II} \\
&\quad + M_1 v_1^2(\mathsf{IV} - \mathsf{III}) + \cdots \qquad (3.4) \\
\frac{1}{R'_\tau} &= \mathsf{IV} - 3\,\mathsf{III} \\
\frac{1}{R'_\sigma} &= \mathsf{IV} - \mathsf{III}.
\end{aligned}
$$

Image given by a perfect system (dark room without lens):

$$l'_1 = -f \tan u_1 = f v_1. \qquad (3.5)$$

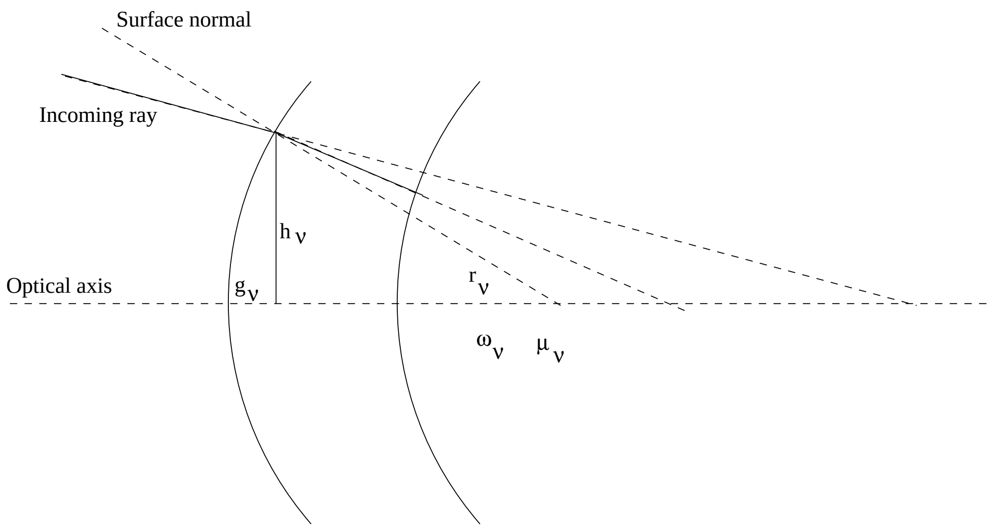
Chapter 4

Calculation of the behaviour of the rays

4.1 Trigonometrical calculation of the functioning of a meridian ray

Initial data:

$u_1; X_1, m_1$	(X_1 calculated starting
$h_1 \approx m_1 + X_1 \tan u_1$	from relation 3.3[XREF!])



4.1.1 The diopter or the mirror is rotation symmetric

Whose profile is given by $g_\nu = g_\nu(h_\nu^2)$ which can usually be developed in the form:

$$g_\nu = a_\nu h_\nu^2 + b_\nu h_\nu^4 + \cdots .$$

Presumed known: u_ν, h_ν.
We calculate for $\nu = 1, 2, 3, \cdots, k$:

$$\begin{aligned} g_\nu &= g_\nu(h_\nu^2) \quad [= (a_\nu + b_\nu h_\nu^2 0 \cdots) h_\nu^2 \;] \\ \tan\omega_\nu &= \frac{dg_\nu}{dh_\nu} \quad [= (2a_\nu + 4b_\nu h_\nu^2 + \cdots) h_\nu \;]; \end{aligned} \tag{4.1}$$

Depending on if we have diopter or mirror:

in the case of a diopter:

$$\begin{aligned} i_\nu &= \omega_\nu - u_\nu \\ \sin i'_\nu &= \frac{n_\nu}{n'_\nu} \sin i_\nu \\ u'_\nu &= \omega_\nu - i'_\nu = u_{\nu+1} \end{aligned}$$

Calculation by successive approximations:

$$\begin{cases} h_{\nu+1} &= h_\nu - (-g_\nu + e'_\nu + g_{\nu+1}) \tan u_{\nu+1} \\ g_{\nu+1} &= g_{\nu+1}(h^2_{\nu+1}); \end{cases} \tag{4.2}$$

in the case of a mirror:

$$u'_\nu = -2\omega_\nu + u_\nu = u_{\nu+1}$$

Calculation by successive approximations:

$$\begin{cases} h_{\nu+1} &= h_\nu - (g_\nu + e'_\nu + g_{\nu+1}) \tan u_{\nu+1} \\ g_{\nu+1} &= g_{\nu+1}(h^2_{\nu+1}); \end{cases}$$

what gives finally:

$$l'_k = h_k - (s'_k - g_k) \tan u'_k. \tag{4.3}$$

4.1.2 The diopter or the mirror is spherical (the radius of curvature not being very large)

Starting point of calculation:

$$\begin{aligned} (\sin i_1)_0 &= (X_1 \varrho_1 - 1) \sin u_1 \\ \sin i_1 &= (\sin i_1)_0 + m_1 \varrho_1 \cos u_1 \end{aligned} \tag{4.4}$$

or well:

$$\sin i_1 = (x_1 \sin u_1 + z_1 \cos u_1) \varrho_1,$$

where x_1, z_1 are the unspecified punctual coordinates of the incidental ray, compared to the center of curve of surface $\nu = 1$.

Presumed known: $i'_{\nu-1}$; $u'_{\nu-1} = u_\nu$.

We calculate for $\nu = 2, 3, \cdots, k$:

for refraction:

$$I'_{\nu-1} = r_{\nu-1} \varrho_\nu$$

for reflection:

$$I'_{\nu-1} = - r_{\nu-1} \varrho_\nu \tag{4.5}$$

for both:

$$\begin{aligned} U'_{\nu-1} &= I'_{\nu-1} - \varrho_\nu e'_{\nu-1} - 1 \\ \sin i_\nu &= I'_{\nu-1} \sin i'_{\nu-1} + U'_{\nu-1} \sin u'_{\nu-1}; \end{aligned}$$

Figure 4.2: figure 3

FIGURE 3 FIGURE 3 FIGURE

$$\begin{aligned}
&\text{diopter:} \\
&\sin i'_\nu = \frac{n_\nu}{n'_\nu}\sin i_\nu \\
&u'_\nu = u_\nu + i_\nu - i'_\nu = u_{\nu+1}; \\
&\text{mirror:} \\
&i'_\nu = i_\nu \\
&u'_\nu = - 2i_\nu - u_\nu,
\end{aligned} \tag{4.6}$$

what gives, in final term:

$$i'_k = [r_k \sin i'_k + (r_k - s'_k)\sin u'_k] : \cos u'_k. \tag{4.7}$$

4.2 Calculation of the behaviour of an arbitrary ray

Starting data:

ξ_1, η_1, ζ_1	consinus direct of the incidental luminous ray,
X_1, M_1, m_1	coordinate point where the ray bores the pupil of input, compared to the node of the 1st surface.

4.2.1 The diopter or the mirror is a surface of revolution

whose profile is given by:

$$g_\nu = g_\nu(h_\nu^2)$$

which can usually be developed in the form:

$$g_\nu = a_\nu h_\nu^2 + b_\nu h_\nu^4 + \cdots.$$

Presumed known:

$g_{\nu-1}, y'_{\nu-1}, z'_{\nu-1}$	co-ordinates of the point $P_{\nu-1}$ where the light ray bores surface $(\nu - 1)$, compared to the node,
$\xi'_{\nu-1}, \eta'_{\nu-1}, \zeta'_{\nu-1}$	cosine direct of the refracted or reflected ray.

Used variables:

$\xi_{\nu n}, \eta_{\nu n}, \zeta_{\nu n}$	the cosine directors of the normal on surface ν at point P_ν
t_ν	the $P_{\nu-1}\,P_\nu$ distance.

The problem consists in calculating co-ordinates g_ν, y'_ν, z'_ν of point P_ν and the cosine directors $\xi'_\nu, \eta'_\nu, \zeta'_\nu$ of the refracted or reflected ray.

To begin calculation one poses:

$$g_0 = 0, \quad e'_0 = -X_1, \quad y'_0 = M_1, \quad z'_0 = m_1.$$

We calculate for $\nu = 1, 2, 3, \cdots, k$:

For refraction:

$$\begin{aligned} y_\nu &= y'_{\nu-1} \\ \xi_\nu &= \xi'_{\nu-1} \\ \eta_\nu &= \eta'_{\nu-1} \\ \zeta_\nu &= \zeta'_{\nu-1} \\ z_\nu &= z'_{\nu-1} \\ t_\nu &= (-g_{\nu-1} + e'_{\nu-1} + g_\nu) : \xi_\nu \end{aligned} \tag{4.8}$$

For reflection:

$$\begin{aligned} y_\nu &= -\ y'_{\nu-1} \\ \xi_\nu &= -\ \xi'_{\nu-1} \\ \eta_\nu &= -\ \eta'_{\nu-1} \\ \zeta_\nu &= \zeta'_{\nu-1} \\ z_\nu &= z'_{\nu-1} \\ t_\nu &= (g_{\nu-1} + e'_{\nu-1} + g_\nu) : \xi_\nu \end{aligned}$$

Iterate these four with successive approximations:

$$\left\{ \begin{aligned} y'_\nu &= y_\nu + t_\nu \eta_\nu \\ z'_\nu &= z_\nu + t_\nu \zeta_\nu \\ h_\nu^2 &= y'^2_\nu + z'^2_\nu \\ g_\nu &= g_\nu(h_\nu^2) \qquad [\, = (a_\nu + b_\nu h_\nu^2 + \cdots) h_\nu^2 \,] \end{aligned} \right.$$

Continuing:

$$\begin{aligned} g'_\nu &= \frac{dg_\nu}{dh_\nu} \qquad [\, = (2a_\nu + 4b_\nu h_\nu^2 + \cdots) h_\nu \,] \\ \xi_{\nu n} &= \frac{1}{\sqrt{1 + g'^2_\nu}} \\ \eta_{\nu n} &= -\ \frac{g'_\nu \xi_{\nu n}}{h_\nu} y'_\nu \\ \zeta_{\nu n} &= -\ \frac{g'_\nu \xi_{\nu n}}{h_\nu} z'_\nu ; \end{aligned} \tag{4.9}$$

For dioptre:

$$\frac{n_\nu}{n'_\nu} \quad ; \; 1-\left(\frac{n_\nu}{n'_\nu}\right)^2$$

$$\begin{aligned}
\cos i_\nu &= \xi_\nu \xi_{\nu n} + \eta_\nu \eta_{\nu n} + \zeta_\nu \zeta_{\nu n} \\
\cos i'_\nu &= \sqrt{1-\left(\frac{n_\nu}{n'_\nu}\right)^2 + \left(\frac{n_\nu}{n'_\nu}\cos i_\nu\right)^2} \\
k_\nu &= \cos i'_\nu - \frac{n_\nu}{n'_\nu}\cos i_\nu \\
\xi'_\nu &= \frac{n_\nu}{n'_\nu}\xi_\nu + k_\nu \xi_{\nu n} \\
\eta'_\nu &= \frac{n_\nu}{n'_\nu}\eta'_\nu + k_\nu \eta_{\nu n} \\
\zeta'_\nu &= \frac{n_\nu}{n'_\nu}\zeta + k_\nu \zeta_{\nu n};
\end{aligned} \tag{4.10}$$

(verification:

$$\xi'^2_\nu + \eta'^2_\nu + \zeta'^2_\nu = 1)$$

For mirror:

$$\begin{aligned}
k_\nu &= -2(\xi_\nu \xi_{\nu n} + \eta_\nu \eta_{\nu n} + \zeta_\nu \zeta_{\nu n}) \\
\xi'_\nu &= \xi_\nu + k_\nu \xi_{\nu n} = -\,\xi_{\nu+1} \\
\eta'_\nu &= \eta_\nu + k_\nu \eta_{\nu n} = -\,\eta_{\nu+1} \\
\zeta'_\nu &= \zeta_\nu + k_\nu \zeta_{\nu n} = \quad \zeta_{\nu+1}
\end{aligned}$$

(verification:

$$\xi'^2_\nu + \eta'^2_\nu + \zeta'^2_\nu = 1)$$

and finally:

$$\begin{aligned}
t'_l &= (s' - g_k) : \xi'_k \\
\Delta L'_k &= y'_k + t'_k \eta'_k \\
\Delta l'_k &= z'_k + t'_k \zeta'_k - l'_I.
\end{aligned} \tag{4.11}$$

If it is about an aspheric surface, we calculate distances t_ν by successive approximations as has already been said. So on the other hand surfaces are spherical (see 422 [XREF!]) one can calculate these distances directly starting from the equation:

$$x_\nu'^2 + y_\nu'^2 + z_\nu'^2 = r_\nu^2,$$

in which $x_\nu'/y_\nu'/z_\nu'$, are the co-ordinate of point P_ν along the optical axis compared to the center of curve O_ν of surface ν. It is not done not need to write the cosine directors of the normal; they are simply represented by: $\xi_{\nu n} = -x_\nu' \varrho_\nu, \quad \eta_{\nu n} = -y_\nu' \varrho_\nu, \quad \zeta_{\nu n} = -z_\nu' \varrho_\nu,$ and we can join together ϱ_ν and constant k_ν in a new $\chi_\nu = -k_\nu \varrho_\nu$ constant. However, if the radius of the surface curvature is quite large, it is appropriate to calculate the distances iteratively, even if surface were spherical.

4.2.2 The diopter or the mirror is spherical (the radius of curvature not being very large)

Presumed known:

$x_\nu,\ y_\nu,\ z_\nu$	co-ordinates of the $P_{\nu-1}$ point where the ray borers surface $(\nu - 1)$, compared to the center of curve of surface ν,
$\xi_\nu,\ \eta_\nu,\ \zeta_\nu$	cosine directors of the luminous ray falling on surface ν.

The geometrical interpretation of the symbols used is given in figure 3 [XREF!].

We calculate:

$$\begin{aligned} A_\nu &= x_\nu \xi_\nu + y_\nu \eta_\nu + z_\nu \zeta_\nu \\ \lambda_\nu &= A_\nu^2 - x_\nu^2 - y_\nu^2 - z_\nu^2 \\ B_\nu &= \sqrt{\lambda_\nu + r_\nu^2} \qquad\qquad (B_\nu \varrho_\nu > 0) \\ t_\nu &= -A_\nu - B_\nu \\ x_\nu' &= x_\nu + t_\nu \xi_\nu \\ y_\nu' &= y_\nu + t_\nu \eta_\nu \\ z_\nu' &= z_\nu + t_\nu \zeta_\nu \ ; \end{aligned} \tag{4.12}$$

for diopter:

$$
\begin{aligned}
c'_\nu &= -r_\nu + e'_\nu + r_{\nu+1}; && \frac{n_\nu}{n'_\nu}; \quad \left(\frac{n_\nu}{n'_\nu}\right)^2 \\
B'_\nu &= \sqrt{\left(\frac{n_\nu}{n'_\nu}\right)^2 \lambda_\nu + r_\nu^2} && (B'_\nu \varrho_\nu > 0) \\
\chi_\nu &= \frac{\frac{n_\nu}{n'_\nu} B_\nu - B'_\nu}{r_\nu^2} \\
\xi'_\nu &= \frac{n_\nu}{n'_\nu} \xi_\nu + \chi_\nu x'_\nu \ = \xi_{\nu+1} && x_{\nu+1} = x'_\nu - c'_\nu \\
\eta'_\nu &= \frac{n_\nu}{n'_\nu} \eta_\nu + \chi_\nu y'_\nu \ = \eta_{\nu+1} && y_{\nu+1} = y'_\nu \\
\zeta'_\nu &= \frac{n_\nu}{n'_\nu} \zeta_\nu + \chi_\nu z'_\nu \ = \zeta_{\nu+1} && z_{\nu+1} = z'_\nu
\end{aligned}
\tag{4.13}
$$

for mirror:

$$
\begin{aligned}
c'_\nu &= r_\nu + e'_\nu + r_{\nu+1} \\
\chi_\nu &= \frac{2B_\nu}{r_\nu^2} \\
\xi'_\nu &= \xi_\nu + \chi_\nu x'_\nu \ = -\xi_{\nu+1} && x_{\nu+1} = -x'_\nu - c'_\nu \\
\eta'_\nu &= \eta_\nu + \chi_\nu y'_\nu \ = -\eta_{\nu+1} && y_{\nu+1} = -y'_\nu \\
\zeta'_\nu &= \zeta_\nu + \chi_\nu z'_\nu \ = \quad \zeta_{\nu+1} && z_{\nu+1} = \quad z'_\nu
\end{aligned}
$$

Part II

Systems consisting of corrector glass and concave mirror

Chapter 5

Suppression of the spherical aberration of a concave mirror by a corrector glass

It is well-known that the spherical aberration of an optical system can be removed by means of a thin with the about parallel faces but suitably deformed blade. To calculate the profile of such a glass placed at the center of curve of a spherical mirror (known as Schmidt system), several methods were proposed. In this work I will present the formulas for a more general case, namely, to cancel the spherical aberration of a concave mirror of a general form by a corrector glass placed at an unspecified distance in front of the mirror (object at infinity).

The profile of a corrector glass can be calculated obviously point-to-point by equalizing the optical paths for all the light rays parallel to the optical axis. In the same way one can calculate the direction of the normal on the surface deformed at each point. The formulas necessary will be given to P1.2[XREF](5.2 ?)

However it is often practical to develop the deformation (g_1) in series and a development until the terms of the 12th degree will be established in P1.3[XREF!](5.3 ?).

Figure 5.1: figure 4
FIGURE 4 FIGURE 4 FIGURE

5.1 Definitions and conventions

Assume the object to be at infinity, the deformed surface of the corrector glass ($\nu = 1$) turned towards the mirror, other surface being plane ($\nu = 0$).

Used variables:

n the index of the corrector glass
e the distance from corrector glass to mirror
$\overline{h}$ the height of the zone of null deviation of the glass
r the radius of curvature (positive) of a concave mirror ($\nu = 2$) along the optical axis
$\overline{r}$ the length of the normal to the mirror at point $\overline{P}_2$ the height hhh (see fig. 4)
$\Delta\varrho$ difference $\varrho - \overline{\varrho}$
$g_{\overline{r}}$ (positive) quantity g corresponding with a sphere of radius of curvature $\overline{r}$
s' the distance mirror-hearth-image
$g_2 = g_{\overline{r}} + \frac{1}{2}\Delta\varrho\left(1 - \frac{h_2^2}{2\overline{h}^2}\right)h_2^2$ profile of the concave mirror

Surlignons quantities relating to the height of the zone of null deviation.

5.1.1 Rigorous calculation of quantities g_1 and g_1'

For the height of the null deviation zone one obtains easily on figure 4:

$$\begin{aligned}
\overline{g}_{\overline{r}} &= \overline{r} - \sqrt{\overline{r}^2 - \overline{h}^2} \\
\overline{g}_2 &= \overline{g}_{\overline{r}} + \tfrac{1}{2}\overline{r}^2 - \overline{h}^2 \\
s' &= \frac{\frac{1}{2}\overline{r}^2 - \overline{h}^2}{\sqrt{\overline{r}^2 - \overline{h}^2}} + g_2.
\end{aligned} \tag{5.1}$$

To calculate the behaviour of an emerging ray of the hearth (primary?) we obtain then:

common part:

$$\begin{aligned} g_{\overline{r}} &= \overline{r} - \sqrt{\overline{r}^2 - h_2^2} \\ g'_{\overline{r}} &= \frac{h_2}{\sqrt{\overline{r}^2 - h_2^2}} \\ g_2 &= g_{\overline{r}} + \tfrac{1}{2}\Delta\varrho \left(1 - \frac{h_2^2}{2\,\overline{h}^2}\right) h_2^2 \end{aligned}$$

first alternate:

$$\begin{aligned} \tan\omega_2 &= g'_2 \; = \; g'_{\overline{r}} + \Delta\varrho \left(1 - \frac{h_2^2}{\overline{h}^2}\right) h_2 \\ \tan u'_2 &= \frac{h_2}{s' - g_2} \\ u_2 &= -2\omega_2 + u'_2. \end{aligned} \tag{5.2}$$

second alternate:

$$\tan u_2 = \frac{h_2(1 - {g'_2}^2) - 2g'_2(s' - g_2)}{2g'_2 h_2 + (1 - {g'_2}^2)(s' - g_2)}.$$

In order to remove the spherical aberration, the optical paths of all the rays parallel with the optical axis must be made equal to that of the axial ray. This leads to:

$$\begin{aligned} \left(n - \sqrt{1 + \tan^2 u_2}\right) g_1 = \quad & e + s' - \sqrt{h_2^2 + (s' - g_2)^2} \\ & -(e - g_2)\sqrt{1 + \tan^2 u_2}. \end{aligned} \tag{5.3}$$

The corresponding height on deformed surface is then:

$$h_1 = h_2 + (-g_1 + e - g_2)\tan u_2. \tag{5.4}$$

The direction of the normal on the deformed surface, corresponding to this height, can be calculated starting from the relation:

$$\tan\omega_1 = g'_1 = -\frac{\sin u_2}{n - \cos u_2} = -\frac{\tan u_2}{n\,\sqrt{1 + \tan^2 u_2} - 1}, \tag{5.5}$$

relation which one easily deduces starting from the condition:

$$n \sin i_1 = n \sin \omega_1 = \sin(\omega_1 - u_2).$$

In particular we find, that with $h_1 = \overline{h}$, equation 5.3[XREF!] can be written as:

$$(n-1)\overline{g}_1 = s' + \overline{g}_2 - \frac{\frac{1}{2}\overline{r}^2}{\sqrt{\overline{r}^2 - \overline{h}^2}} = -\left(\frac{\overline{g}_{\overline{r}}^2}{\sqrt{\overline{r}^2 - \overline{h}^2}} - \frac{1}{2}\Delta\varrho\overline{h}^2\right).$$

The focal length of the system can be calculated by the formula:

$$f = s'\left[1 + e(\sigma' - 2\varrho)\right] = e - s'(2e\varrho - 1). \tag{5.6}$$

In the particular case of a spherical mirror ($r = \overline{r}$) these formulas can be simplified somewhat. With the introduced symbols the general formula of reflection for a spherical surface can be written as:

$$\frac{1}{\varrho + \sigma_2} + \frac{1}{\varrho - \sigma_2'} = 2g_2, \tag{5.7}$$

what will make it possible to calculate (redoing equation 5.1[XREF!]):

$$\begin{aligned} s' &= r - \frac{\frac{1}{2}r^2}{\sqrt{r^2 - \overline{h}^2}} = \frac{1}{\sigma'} \\ K &= \frac{0.5}{\sigma' - \varrho} = \frac{1}{2}r - \overline{g}_2 \\ \sigma_2 &= -\varrho + \frac{0.5}{g_2 + K}, \end{aligned}$$

and then (redoing equation 5.2[XREF!]):

$$\tan u_2 = \frac{h_2}{s_2 + g_2} = \frac{h_2\sigma_2}{1 + g_2\sigma_2}.$$

TODO PHASE: Subsection 1.3, page 23.

Part III

Anastigmatic telescopes with field corrector lenses

Bibliography

[1] BERNHARD SCHMIDT, Ein lichtstarkes komafreies Spiegelsystem, Central-Zeitung für Optik und Mechanik, 52. Jahrgang, Heft 2, 1931.

[2] Y. VÄISÄLÄ, Anastigmatisches Spiegelteleskop, der Sternwarte der Universität Turku, Astr. Nachr. *254*, 361, 1935.

[3] Y. VÄISÄLÄ, Über Spiegelteleskope mit grossem Gesichtsfeld, Astr. Nachr. *259*, 197, 1936

[4] F. B. WRIGHT, An aplanatic reflector with a flat field, related to the Schmidt telescope, Publ. Astr. Soc. Pac. *47*, 300, 1935

[5] HORST KÖHLER, Die Entwickling der aplanatischen Spiegelsysteme, Astr. Nachr. *278*, 1 – 14, 1949.

[6] KAYE E. WEEDON, (Ramstad près d'Oslo, Norvège), A Bibliography on Schmidt Cameras and Related Mirror Optical Systems. Microfilm montrant la bibliographie jusqu'au 22 novembre 1949.

[7] Y. VÄISÄLÄ, Om prisbilliga jätteteleskop, Pop. Astr. Tidskrift nr. 3 – 4, 1949.

[8] Y. VÄISÄLÄ,Nuevo procedimiento para construir telescopios gigantes, Urania num. 220, 1949.

[9] E. H. LINDFOOT, On the Optics of the Schmidt Camera, M. N., *109*, 3, 1949.

[10] KARL STREHL, Über Luftschlieren und Zonenfehler, Zeitschr. f. Instrk. 22, p. 213, 1902.

[11] Y. VÄISÄLÄ, Neue Methoden zur Untersuchung der Objektive nebst Bemerkungen über die Beurteilung ihrer Güte, Ann. Univ. Turkuensis N:o 2, 1922.

[12] B. STRÖMGREN, Das Schmidtsche Spiegelteleskop, Viertahljahrsschr. d. Astron. Gesellschaft, 70, 1935.

[13] HANS LEHMANN, Anwendung der Hartmannscen Methode der Zonenprüfung auf astronomische Objektive, Zeitschr. f. Instrk. 22, p. 108, 1902.

[14] E. H. LINDFOOT, On the aberrations of the Field-flattened schmidt camera, M. N., *109*, 5, 1949.

[15] H. SIEDENTOPF, Grundriss der Astrophysik, 1950

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